What One Can Learn About the $e^+ + e^-$ Energy Spectrum from Radio Observations

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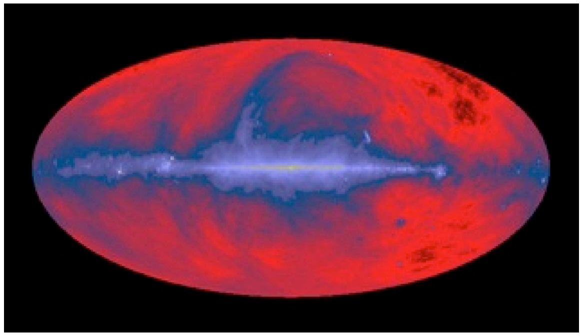
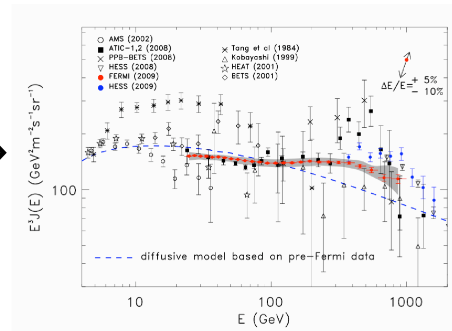
Fermilab
10/8/09

Radio Observations Can Map $e^{+} + e^{-}$ Spatial/Energy Distribution



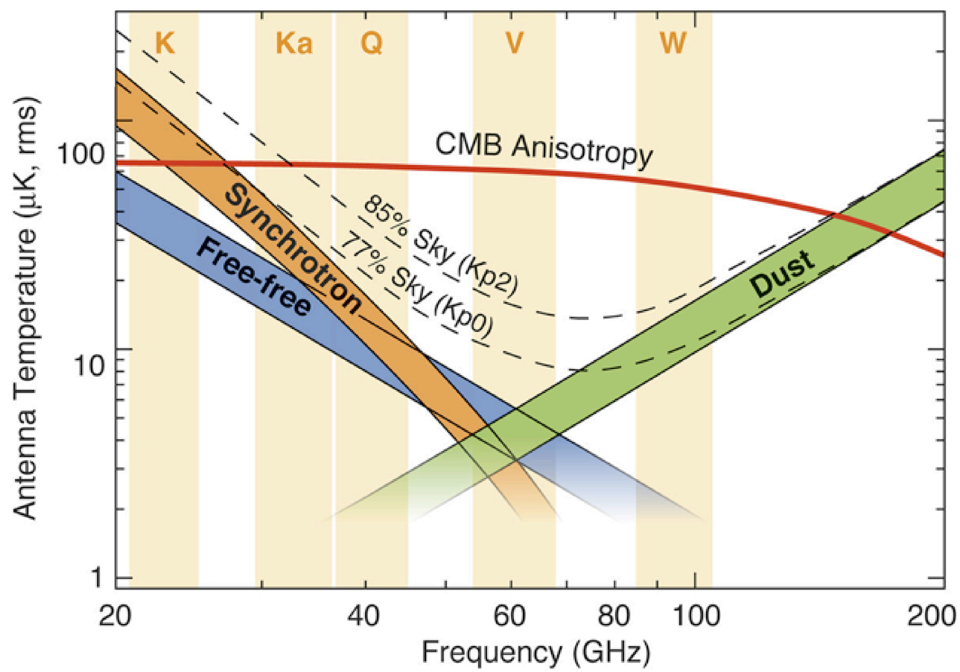
Galactic Synchrotron Emission (Haslam)



(Local) e^{+}/e^{-} Energy Spectrum

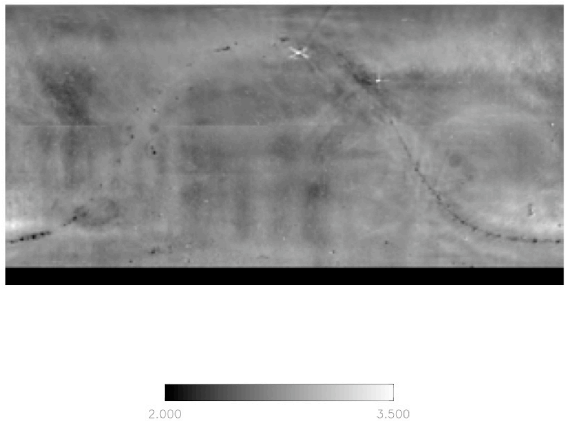
But need spectral information for the radio emission

Many Sources of Emission



Radio observations least contaminated

Spectral Information Fairly Limited



Spectral Index map (Platania 2003)

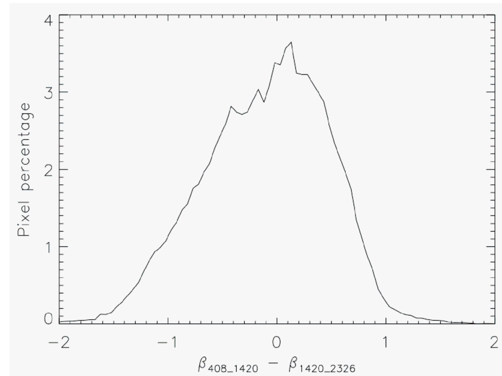


Fig. 8. Distribution of the differences $\beta_{408,1420} - \beta_{1420,2326}$. The binsize is 0.01.

Spectral Curvature (Platania 2003)

$T_{\mathrm{RJ}} \propto \nu^{-\beta}$ or $I_\nu \propto \nu^{2-\beta}$

Global Modelling / Template Fitting

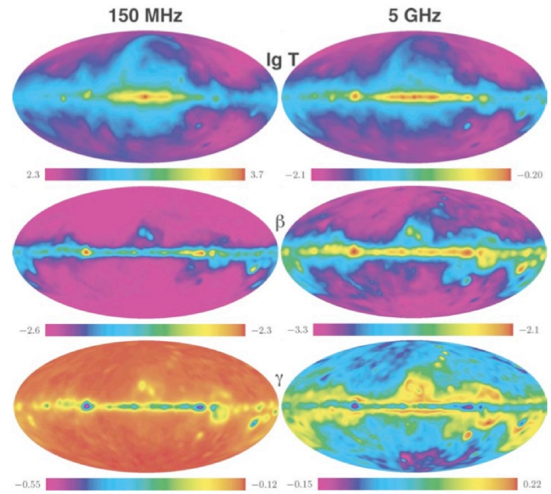
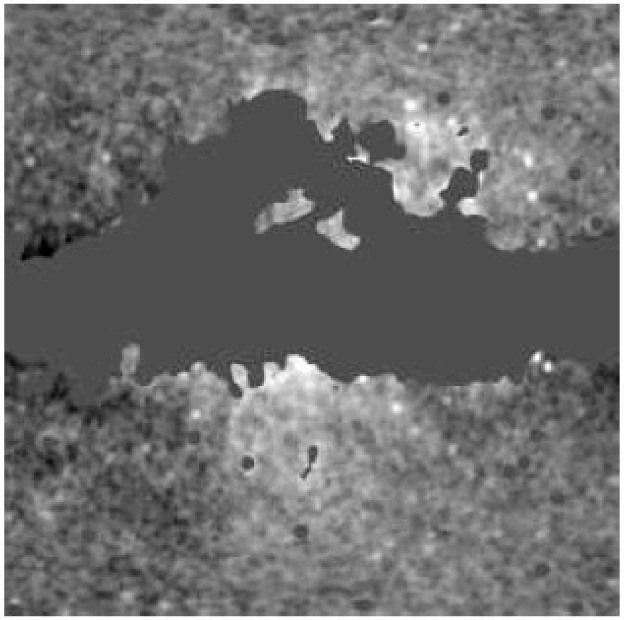


Figure 10. Sky maps (from top to bottom) of the temperature, spectral index β, and 'the running' (or variation) of spectral index γ at 150 MHz (shown on the left-hand side) and 5 GHz (shown on the right). Whereas the 150-MHz emission is dominated by synchrotron radiation with a spectrum that is both falling ($\beta \sim -2.5$) and steepening ($\gamma < 0$), the 5-GHz emission has a much broader range of spectral indices that are mostly getting less negative towards higher frequency ($\gamma > 0$).

Oliveira–Costa *et al.* 2008

But modelling is not really measurement but rather interpolation with assumptions.

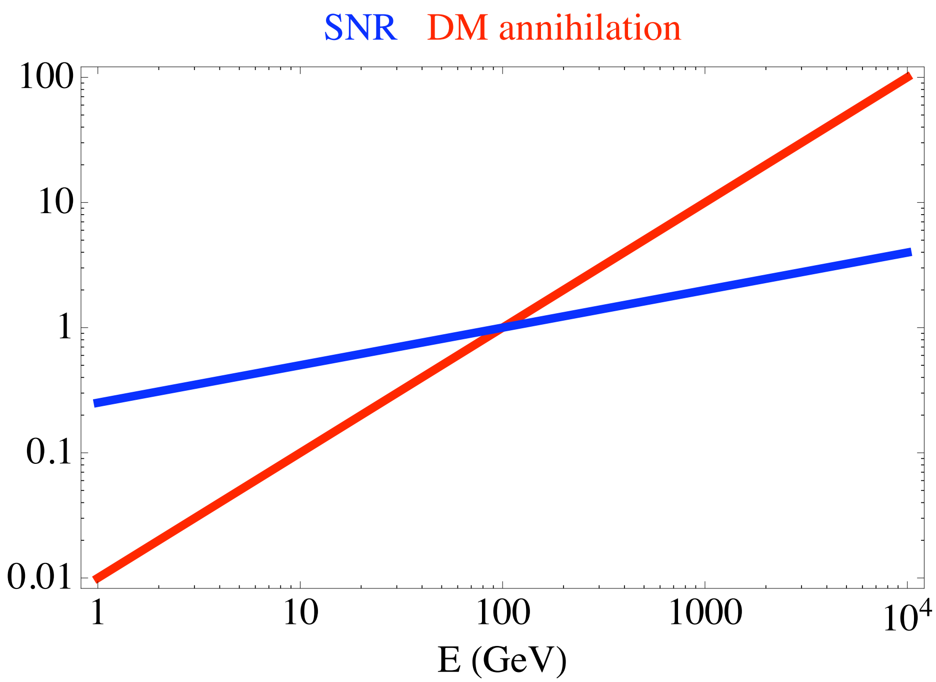
WMAP Haze



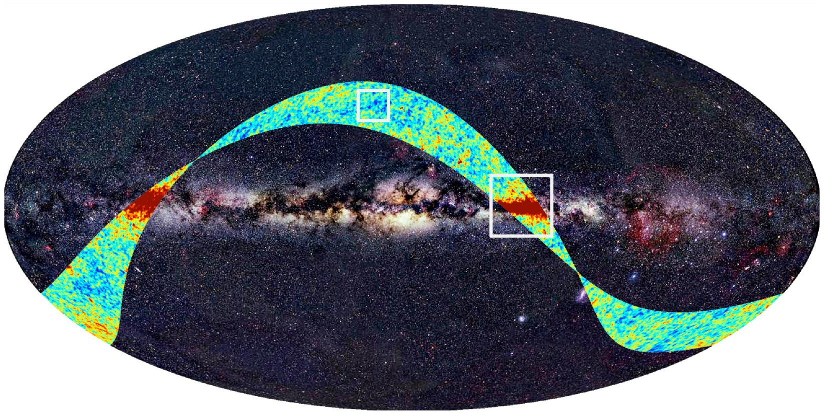
WMAP Haze

Gotten by subtracting out "normal" foregrounds including "generic" synchrotron.

WMAP Haze



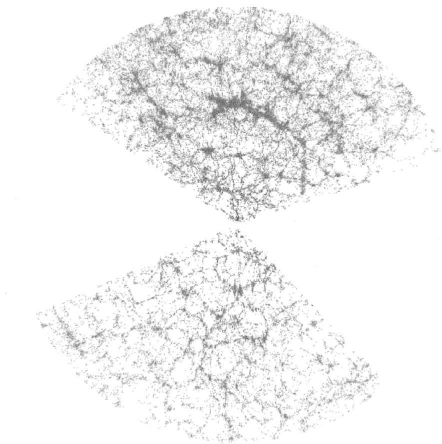
Better Spectra for CMB Arriving



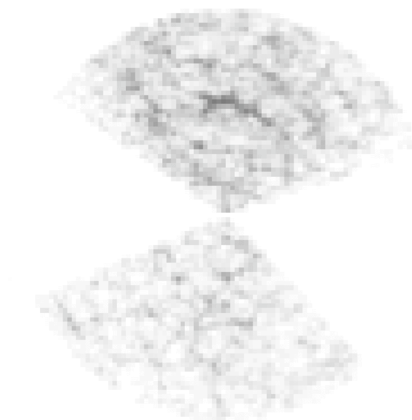
PLANCK First Light Survey 9/09

PLANCK has 10 frequency channels! Data not forthcoming until 2012.

21 cm Redshift Surveys Coming Soon



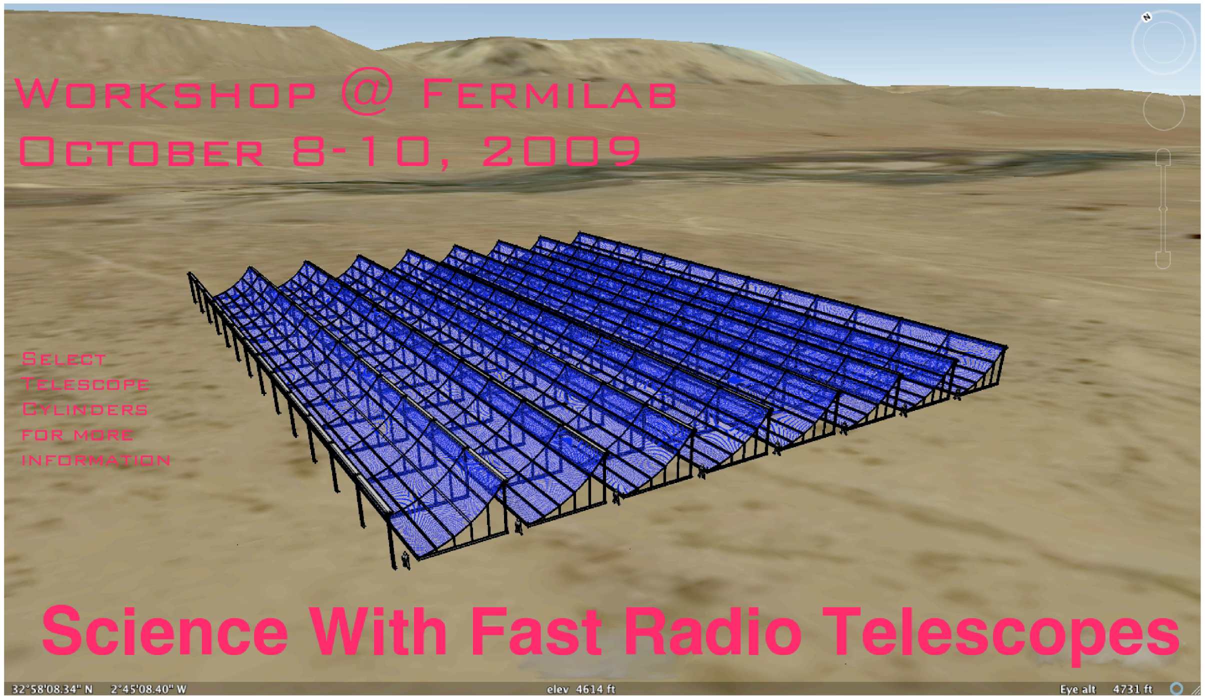
Optical Redshift Survey (SDSS) galaxy by galaxy



21 cm Redshift Survey – intensity mapping

Synchrotron 10,000 times 21cm signal!

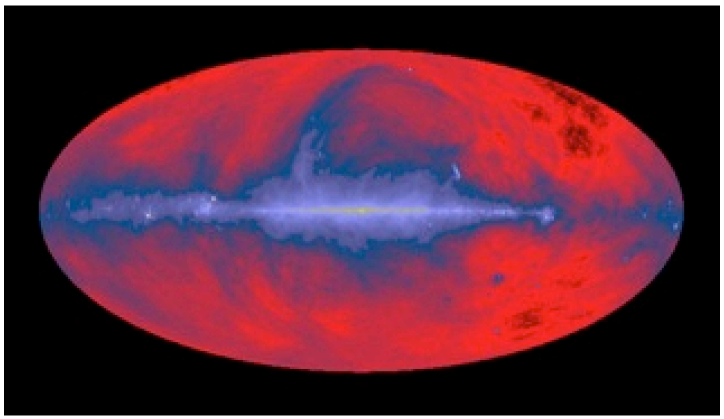
e.g. Cylinder Radio Telescope



$\Delta T \sim 100\,\mu K\,/\,\text{pixel},\ \delta\nu \sim 1\,\text{MHz},\ \Delta\nu \sim 1\,\text{GHz},\ \nu \sim 1\,\text{GHz},\ \delta\theta \sim 10',\ \Delta\Omega \sim 2\,\pi$

Differential Measurements!

Radio meassurements unable to do absolute intensity measurements.
However for very wide field-of-view surveys can hope to difference.
e.g. WMAP Haze region - NCP



Electrons > Synchrotron

The intensity from an isotropic distribution of electrons in a magnetic field is given by the line-of-sight integral

$$I_\nu^{\text{sync}}[\hat{\boldsymbol{n}}] = \frac{\alpha_e}{(2\pi)^2} \int dl \int_0^\infty d\gamma \, \frac{dn_e[\boldsymbol{x}]}{d\gamma} \, \frac{h\nu}{\gamma^2} \, F\left[\frac{4\pi m_e c \nu}{3 e \gamma^2 \left|\hat{\boldsymbol{n}} \times \boldsymbol{B}[\boldsymbol{x}]\right|}\right]$$

where

$\frac{dn_e}{d\gamma}$ – number density of electrons per unit γ

γ – electron energy $(m_e c^2 \text{ units})$

$\alpha_e = \frac{e^2}{\hbar c}$ – fine structure constant

B – magnetic field

$$F[y] \equiv y \int_{-\infty}^{\infty} dX \left(1 + X^2\right)^2 \left(K_{2/3}\left[\frac{1}{2} y \left(1 + X^2\right)^{3/2}\right]^2 + \frac{X^2}{1+X^2} K_{1/3}\left[\frac{1}{2} y \left(1 + X^2\right)^{3/2}\right]^2\right)$$

N.B. The argument to F is $\propto \nu^1 \gamma^{-2}$ so we can invert it by deconvolution.

Synchrotron > Electrons

Mathematically this is a linear transfrom

$I_\nu = M \cdot J_E$

where

J_E – input electron spectrum electron energy

I_ν – output synchrotron spectrum

Naively the inverse is $J_E = M^{-1} \cdot I_\nu$.

To better understand one can analytically "diagonalize" M.

Generalized Convolution

In physics we often find a transform of the form $G[x] \to F[x]$

$$F[x] = A[x] \int_0^\infty dy\; B[y]\, K[x^p\, y^q]\, G[y]$$

where $A, B, K : \mathbb{R} \to \mathbb{C}$, $0 \neq p, q \in \mathbb{R}$, $p, q \neq 0$. Want to find $F[x] \to G[x]$.

Choose another real number r, and then define

$$\eta \equiv p \ln[x] \qquad \zeta \equiv -q \ln[y] \qquad f[\eta] \equiv e^{-r\,\eta} \frac{F\left[e^{\eta/p}\right]}{A\left[e^{\eta/p}\right]} \qquad g[\zeta] \equiv \frac{B\left[e^{-\zeta/q}\right] G\left[e^{-\zeta/q}\right]}{e^{(1/q-r)\zeta}} \qquad k[\xi] \equiv e^{r\,\xi}\, K\left[e^{\xi}\right]$$

With these definitions the transform is simply a convolution

$$f[\eta] = \int_{-\infty}^{\infty} d\zeta\; k[\eta - \zeta]\, g[\zeta].$$

If we Fourier transform $a[x] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dK\, e^{i K x}\, \tilde{a}[K]$ and inverse $\tilde{a}[K] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, e^{-i K x}\, a[x]$ so

$$\tilde{f}[K] = \sqrt{2\pi}\; \tilde{k}[K]\, \tilde{g}[K]\,.$$

Choose r such that $\left|\tilde{k}[K]\right| < \infty$ (usually $\tilde{k}[0] < \infty$ is sufficient).

Inverse Transform

Formally

$$\tilde{g}[K] = \frac{1}{\sqrt{2\pi}\,\tilde{k}[K]}\,\tilde{f}[K]$$

and then Fourier transform back.

But ... usually $\lim_{|K|\to\infty} \tilde{k}[K] = 0$:

if $k[\xi] \in C^{\infty} \Rightarrow \forall\ A[p] < \infty$ such that $\left|\tilde{k}[K]\right| < \frac{A[p]}{|K|^{p}}$

Exponentially Small $\tilde{k}[K] \Rightarrow$ Exponentially Large $\tilde{g}[K]$.

So inverse is "unstable" *i.e.* noise on small scales (large K) is highly amplified.

Need to "regularize" inverse - want smooth $\tilde{g}[K]$.

Since we have diagonalizes in K-space the large and small scales are decoupled!

Decoupled large in K ill-conditioned modes from small K well conditioned modes.

Applications

Mathematics: Laplace Transform: $g[s] = \int_0^\infty f(t)\, e^{-s\,t}\, d\,t$.

Physics:

Planck Transform: $n[\nu] = \int_0^\infty d\,T\, \frac{f[T]}{\mathrm{Exp}\left[\frac{h\,\nu}{k\,T}\right]-1}$.

Non-Relativistic Thermal Free-Free Emission:

$$I_\nu^{\mathrm{ff}}[\hat{\boldsymbol{n}}] = \frac{4\sqrt{2}}{3\sqrt{\pi}}\, \frac{e^6}{(m_e\, c^2)^2} \int_0^\infty d\,T_e \sqrt{\frac{m_e\, c^2}{k_B\, T_e}} \int d\,l\, E_1\left[\frac{4\,\pi^2\, m_e\, \lambda_B{}^2\, \nu^2}{2\,K^2\, k_B\, T_e}\right] Z_{\mathrm{eff}}[l,\, \boldsymbol{x}]\, \frac{d(n_e[\boldsymbol{x}])^2}{d\,T_e}\,.$$

Ultra-Relativistic Synchrotron Emission:

$$I_\nu^{\mathrm{sync}}[\hat{\boldsymbol{n}}] = \frac{\alpha_e}{(2\,\pi)^2} \int_0^\infty d\,\gamma\, \frac{h\,\nu}{\gamma^2} \int d\,l\, F\left[\frac{4\,\pi\, m_e\, c\,\nu}{3\,e\left|\hat{\boldsymbol{n}}\times \boldsymbol{B}[\boldsymbol{x}]\right|\gamma^2}\right] \frac{d\,n_e[\boldsymbol{x}]}{d\gamma}\,.$$

Inverse Compton Scattering

Synchrotron In Fourier Space

Choosing and fiducial numbers ν_{fid}, γ_{fid}, L_{fid} (luminance energy/time/solid angle)

$$\tilde{f}[K] \equiv \int_0^\infty d x \, x \, F[x] \, e^{-i K \ln[x]}$$

$$\tilde{I}\left[K, \hat{n}\right] \equiv \frac{1}{L_{\text{fid}}} \int_0^\infty d\nu \, I_\nu^{\text{sync}}\left[\nu, \hat{n}\right] e^{-i K \ln\left[\frac{\nu}{\nu_{\text{fid}}}\right]}$$

$$\tilde{\Gamma}\left[K, x, \hat{\beta}\right] \equiv \frac{4\pi}{n_{\text{fid}}} \int_0^\infty d\gamma \left(\frac{\gamma}{\gamma_{\text{fid}}}\right)^2 \frac{d n_e}{d^2\hat{\beta}\, d\gamma} e^{-i\, 2 K \ln\left[\frac{\gamma}{\gamma_{\text{fid}}}\right]}$$

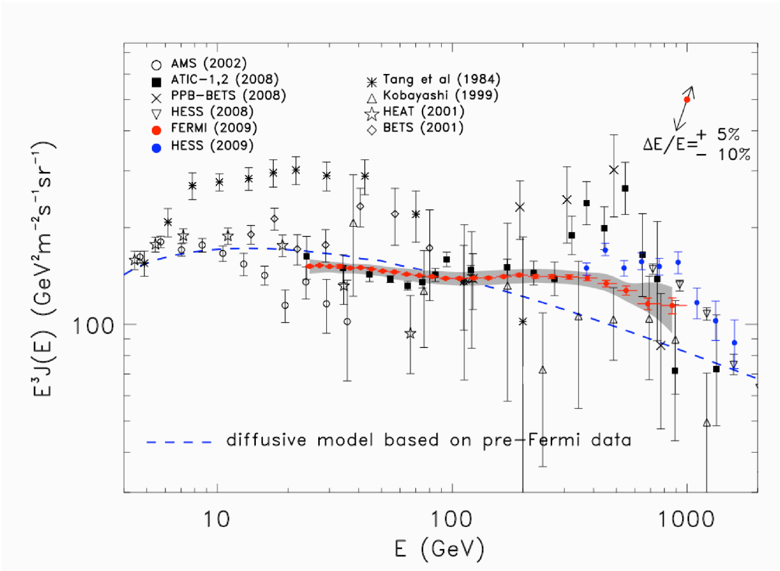
FYI $\tilde{f}[0] = \int_0^\infty d x \, x \, F[x] = \frac{16\pi^2}{27}$, $\tilde{I}\left[0, \hat{n}\right]$ is the total energy flux in all band in units of L_{fid}.

$$\tilde{I}\left[K, \hat{n}\right] = e^{i K \ln\left[y_{\text{fid}}\right]} \frac{\tilde{f}[K]}{\tilde{f}[0]} \int \frac{d l}{l_{\text{fid}}} \left(\frac{B}{B_{\text{fid}}}\right)^2 \text{Cos}[\psi] \, e^{-i K \ln[\text{Cos}[\psi]]} e^{-i K \ln\left[\frac{B}{B_{\text{fid}}}\right]} \int \frac{d^2\hat{\beta}}{4\pi} \tilde{\Gamma}\left[K, x, \hat{\beta}\right] \delta[\psi - \delta]$$

where $y_{\text{fid}} \equiv \frac{4\pi \, m_e \, c \, \nu_{\text{fid}}}{3\,\gamma_{\text{fid}}^2 \, e \, B_{\text{fid}}}$, $l_{\text{fid}} = 3\pi \frac{L_{\text{fid}} \, m_e^2 \, c^3}{e^4 \, n_{\text{fid}} \, B_{\text{fid}}^2 \, \gamma_{\text{fid}}^2}$, $\text{Cos}[\psi] = \frac{|\hat{n}\times B|}{|B|}$, $\text{Sin}[\delta] = \hat{\beta} \cdot \frac{B}{|B|}$.

Lebesgue Integration

Instead of thinking of a line-of-sight integral think of it as an integral over the electron weighted distribution of $p\left[\sqrt{\left|\hat{n}\times B[x]\right|}\ \gamma\right]$

in the 

$E\sqrt{B}\ \left(\text{Gev}\sqrt{\mu G}\right)$

Model Distributions

1) (An)Isotropy $\hat{\beta}$ at each "point":

    assume isotropic

2) (An)Isotropic $\boldsymbol{B}$ at each "point":

    assume istropic

3) Distibution of B at each point:

    assume Gaussian axial vector

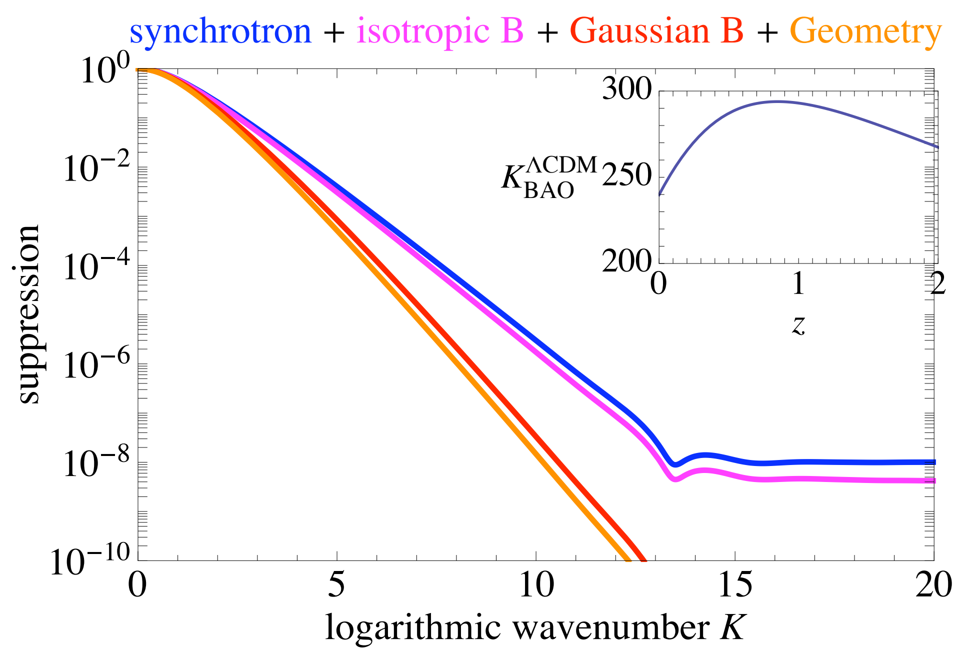
4) Geometric distribution of $\boldsymbol{B}$ and $\frac{dn_e}{d\gamma}$ along line-of-sight:

    assume Gaussian slab: $\frac{dn_e}{d\gamma}$, $B^2 \propto e^{-\frac{z}{z_*}}$

Each a convolution which suppresses I_ν fluctuations!

Synchrotron Suppression

or Why You Can See 21cm LSS



Synchrotron Suppression

$\log_{10}[K]$

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Positivity and Regulation

Lebesgue integral is convolution is with a positive definite kernel

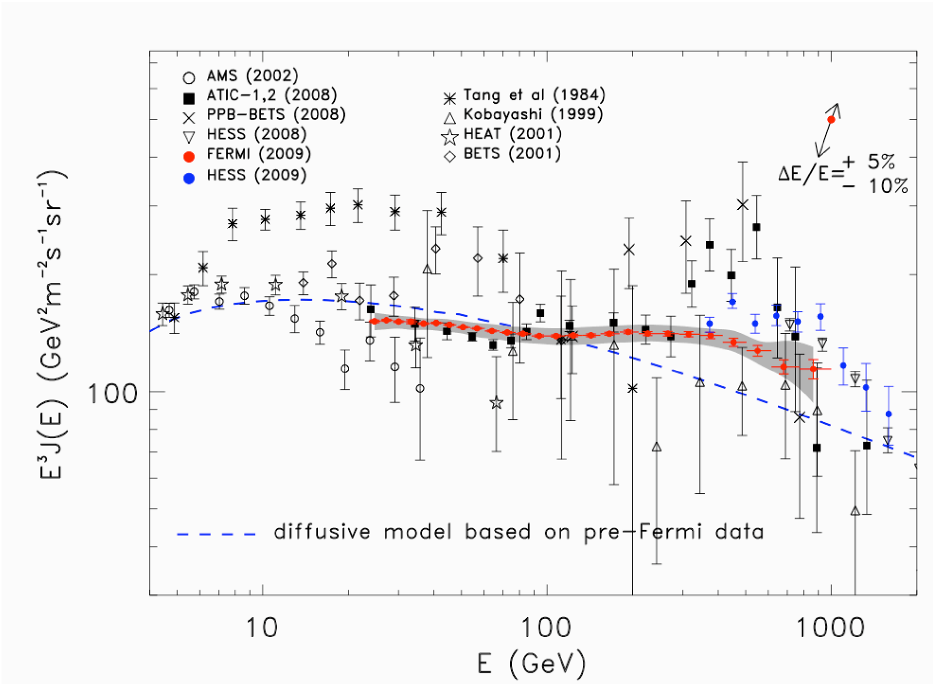
$p[\gamma, \hat{\beta}, B] \geq 0$ line-of-sight:

Large exponential suppression means that high frequency noise in I_ν cannot be multiplied by $\frac{1}{\tilde{k}[K]}$ but must be suppressed.

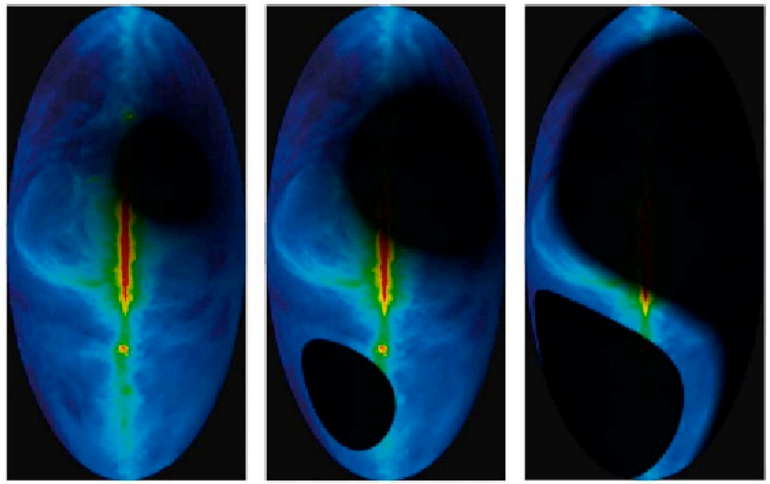
This regulates the inversion process in a way which is not ad hoc like Wiener filtering.

Noisey high frequency components are useless - only small number of spectral modes are useful.

Local Electron/Positron Distribution



Sky Coverage? Moniez



- For $A = 0°$, 28200 square degree (68%) of the sky are covered with a daily exposure larger than 300s, and 1150 square degree (3%) are covered with an exposure larger than 1500s.
- For 45°, 22200 square degree (54%) of the sky are covered with a daily exposure larger than 300s, and 2100 square degree (5%) are covered with an exposure larger than 1500s.
- For 90°, 9600 square degree (23%) of the sky are covered with a daily exposure larger than 300s, and 4200 square degree (10%) are covered with an exposure larger than 1500s.

Conclusions

1) A good model-independent measure of the electron energy distribution would clarify claims of indirect detection of dark matter annihilation products.

2) Non parametric synchrotron → electron transform easy

   a) use K-space to avoid instability

   b) positivity provide natural regularization of inversion

   c) only a relatively small number of lo-K modes contribute in each angular resolution element.

3) one or more additional all sky maps eliminates degeneracies.

4) 21cm intensity mapping redshift surveys will provide such a map

   a) will measure intensity, spectral index, running of index, ...

   b) could map outflow from pulsars

   c) scientific justification to extend frequency reach?

   d) scientific justification for including Galactic center?

Initialization

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